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# Hydraulic Brake Model

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**Project Number: ECC5**

**Hydraulic Brake Model**

**A Major Qualifying Project Report**

**Submitted to the Faculty  
Of the**

**WORCESTER POLYTECHNIC INSTITUTE**

**In partial fulfillment of the requirements for the**

**Degree of Bachelor of Science**

**By**

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**Ryan Moseley**

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**Mikhail Tan**

**Date: April 24, 2013**

**Approved:**

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**Professor Eben C. Cobb**

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## Abstract

The objective of this project was to create a physical model to demonstrate the transfer of forces from the mechanical domain to the hydraulic domain, and back to the mechanical domain. To do this, a spinning wheel was designed to be stopped using a hand brake connected hydraulically to a set of calipers and a brake rotor. A wheel on a dead axle was spun up via a motor and once the wheel was at speed, mechanical force was applied to the hand brake to bring the wheel to a stop. Calculations for the model were made to determine flexure, stress concentrations, natural frequency and the mechanical advantage of the braking system. The calculations showed that under normal operating conditions, the whole assembly will not have any noticeable flexure, the stress concentrations in the axle will not lead to catastrophic failure, the natural frequency of the rotating wheel on a simply supported axle is far above that of the operating frequency, and the hydraulic brake has ample mechanical advantage to safely bring the wheel to a stop. Once all these calculations ensured that the model would not fail during operation, a physical model was constructed. Testing and operation of the physical model showed that the calculations were accurate. The final model successfully shows the transfer of mechanical force (the user squeezing the handle) to hydraulic force (moving a piston) to mechanical force, which generates friction between the calipers and the brake rotor to stop a rotating wheel.



## Introduction

The purpose of the project was to demonstrate the flow of energy from mechanical to fluid back to mechanical. The Goal of the project was to create a working classroom model that will last up to ten years. The mechanical force comes from the operator's hand squeezing the brake lever, which compresses hydraulic fluid. The hydraulic fluid in the line becomes pressurized and pushes on a piston that squeezes the brake pads onto the brake disk. The model will be able to show different braking scenarios by applying differing amounts of force to the handle. For example the motor can be run with minimal braking and the wheel will spin at a slower speed than without braking. Another extreme is stopping the wheel instantaneously by applying rapid firm pressure to the brake handle.

## Background

The first documented case of brakes in use was in ancient Rome. These simple brakes were composed of a lever that when pulled, pressed a wooden block onto the outside of a metal lined wheel. The primary force for braking with this device was friction. This method was effective due to the slow speeds at which the carts traveled; however, it was an inadequate form of slowing runaway carts. This method of braking was used for centuries with little design improvement.

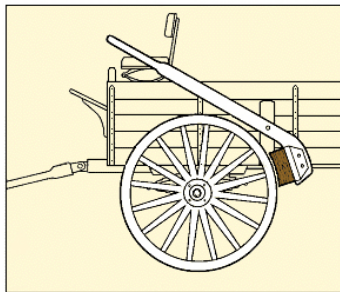


Figure 1: Example of the Lever Brake

When the Michelin brothers created the rubber-covered wheel wooden blocks were replaced with drum brakes. Louis Renault invented drum brakes in 1902. Instead of applying a block to the outside of the wheel, drum brakes were mounted inside of the wheel hubs. This helped minimize debris blockage and reduce the loss in braking friction. Drum brakes are still in use in cars as handbrakes due to the large amount of force needed in order to overcome the brake force while at rest.



Figure 2: Example of a Drum Brake

With the introduction of the assembly line, cars became heavier and faster, which created a need for a more powerful braking system. Malcolm Loughead created a four-wheeled hydraulic braking system. The hydraulic system uses lines filled with hydraulic fluid rather than cable driven braking systems. The main advantage to hydraulic braking systems is that they can apply a greater braking force than cable systems. Cable brakes fatigue faster than hydraulic brakes due to the constant tension that the cable is under. Hydraulic brakes allowed the driver to apply less force onto the brake pedal while still stopping in the same short distance.

Throughout braking history the issue of overheating has been a constant problem. Heat occurs when the brake pads come in contact with the braking surface. The key factor in dispersing heat is having a larger surface area for the brake to cool down. Disk brakes have a large surface area exposed to the air, which helps it to remain cooler. There are holes and grooves cut into the rotor of the braking system to allow water and debris to be moved off the braking surface and minimize interference, which causes loss of braking force.



**Figure 3: Example of a Disk Brake**

Disk brakes did not start becoming popular in vehicles until the 1950's even though they were invented around 1902. Disk brakes are attached inside the rim of the vehicle and spin in unison with the wheel. When force from the driver's foot is applied to the brake pedal the brake fluid travels through hydraulic cables and becomes amplified by the power braking system attached to the engine; this in turn pushes the brake fluid against the caliper which uses frictional force to slow the vehicle. Faster vehicles need brake pads and calipers to be made of different materials to replicate the same braking distance needed to stop slower less advanced vehicles, due to the greater amount of inertia that is trying to be stopped.

There are five main materials used in brake rotors. The five materials most commonly found in brake rotors are cast iron, steel, layered steel, aluminum, and high carbon irons. Production cars use cast iron brakes due to the amount of abuse that they can handle without cracking or failing. Steel brakes have a lighter weight and heat capacity, but lack durability in repeated uses. Heat can disperse faster with layered steel brakes because adding layers to simple steel brakes allows for a

stronger material that can withstand a more rigorous workload. Aluminum brakes have the lowest weight of all vehicle rotors. Heat is dispersed quicker, however the total capacity for heat absorption is lower than in steel brakes; this is why aluminum is most commonly used in motorcycles and other small vehicles. The final type for brake material that is used is high carbon iron. High amounts of carbon allow for increased heat diffusion, which makes this type of brake most commonly used in high performance vehicles.



**Figure 4: Brake Rotors**

Brake pads have been made with different materials throughout the years depending on the intended use. Asbestos was the most popular material due to its ability to absorb and disperse heat. After scientific studies, asbestos has been found to be a highly toxic material and has been banned from use in vehicles in the United States. With asbestos illegal to use, brake manufacturers were forced to create safer brakes from a material that will not harm the general public. Organic brakes are made from materials that can withstand heat, for example; glass and varieties of rubber are mixed with a heat resilient resin to produce safer brakes. The advantages of using organic brake pads are that they are usually quieter and are

easier to dispose. Even so, organic brakes are not typically used because they wear easily and dust particles collect between the pad and wheel, which decreases the braking surface.



**Figure 5: Brake Pads**

With a lighter weight to slow down, motorcycles use organic and ceramic brake pads. Ceramic brake pads are the most effective type of brake pads but are the most costly. The most common type of brake pad is made with a mixture of several types of metals. These metallic brakes are durable while still being cost efficient. The negative factors for using metallic brakes are that they work best when warm and it may take longer to slow down at first when driving in cold weather. With advances in material science, brakes will continue to improve to match the advances in car technology.

## Analysis

Our design was optimized to minimize the amount of material and space that was needed for the model while still keeping the strength and longevity of the physical model.

In order to calculate the natural frequency of the shaft, the weighted diameter was calculated to be 0.591 in. The area second moment of inertia of the shaft equals  $2.887 \times 10^{-7} \text{ ft}^4$ . The mass of the wheel was weighed to be 0.86 lbs. The length and modulus of elasticity of the shaft are 11.25 in and  $3.046 \times 10^4 \text{ ksi}$  respectively.

$$I_{\text{shaft}} := \frac{\pi \cdot D_{\text{shaft}}^4}{64}$$

Second Moment of Area

$$\omega_{\text{shaft}} := \left( \frac{2}{\pi} \right) \left( \frac{3 \cdot E_{\text{steel}} \cdot I_{\text{shaft}}}{\text{Mass} \cdot L_{\text{shaft}}^3} \right)^{\frac{1}{2}}$$

Natural frequency of shaft

The natural frequency of the shaft was calculated to equal 264.4 Hz. The frequency of the motor equated to be 1.667 Hz. This gave the model a frequency comparison from the shaft to motor of 1 to 158.64.

All of the other finite elements were found using Solidworks. The shaft with wheel and rotor was found to be balanced using the center of mass simulation. The deflection, von Mises stress, and safety factor were calculated using the SimulationXpress Study. The deformation was 0.083 mm. The von Mises stress

equated to equal  $8.29 \times 10^7 \text{ N/m}^2$ . The minimum safety factor was 7.48, which exceed the recommended mechanical safety factor of 4. These simulations were run with solid works due to the complicity of the shaft and loading.

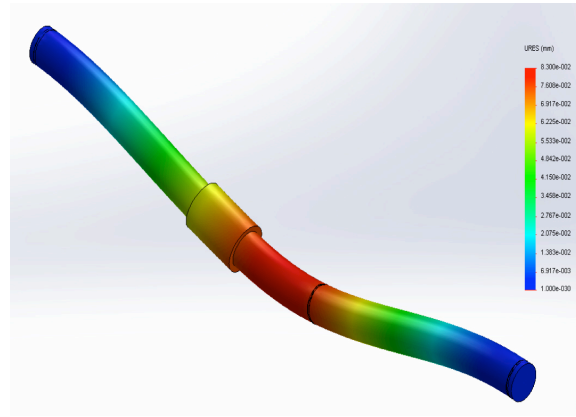


Figure 6: SolidWorks simulation of Deformation

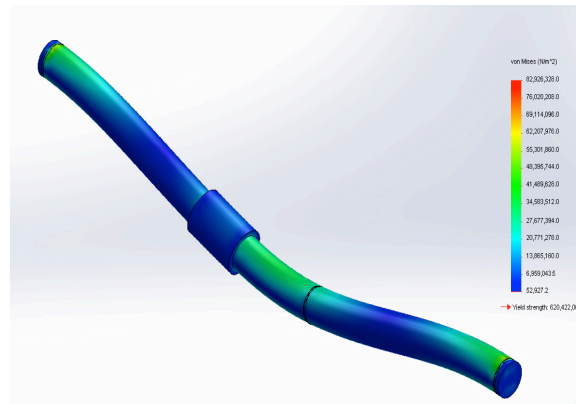


Figure 7: SolidWorks simulation of von Mises Stress

To calculate the mechanical advantage in the brake, we used a torque wrench to spin the rotor while a fixed weight hung from the brake lever. When the brake slipped, the torque wrench was applying more force than the brake handle. The



torque that equaled the 13.75 lbs that hung from the handle was 30 lb\*ft.

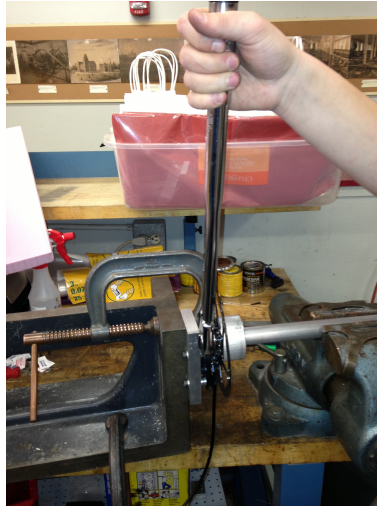


Figure 8: Testing to Find the Mechanical Advantage of the Hydraulic Brake

$$AMA := \frac{F_{out}}{F_{in}}$$

Mechanical Advantage

When taking any part from a solid model to a solid piece of metal, there are two concerns that face any machinist. The first problem is fixturing. Fixturing is how any piece of metal is held in a machine. The second problem is the tolerance. If a part is out of the stated tolerance, it can cause major problems in an assembly when products are being built.

## Methodology

When the project was first presented, the group was given a preliminary sketch of what the desktop model was going to look like. This simple sketch involved a piece of 90 degree angled metal that would support one side of a shaft. The other side of the shaft would be supported by a simple piece of metal thus creating a simply supported beam. The flywheel that was proposed would be a common wheelbarrow wheel with a bearing already pressed in place. The flywheel would then be electrically spun and stopped using a commercially available brake. With this brief sketch, the second revision of the assembly was created in SolidWorks.

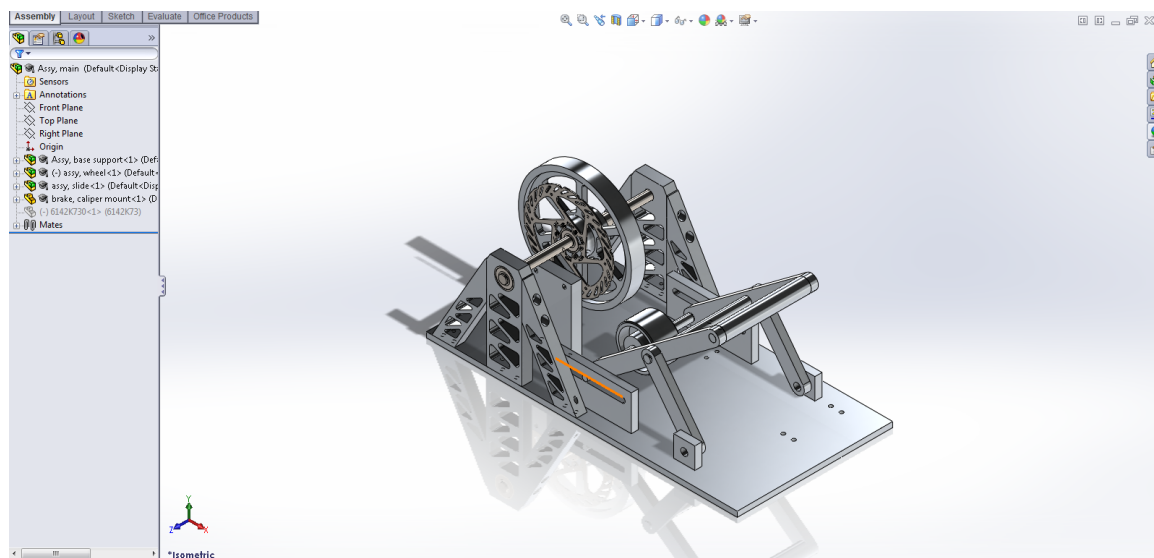


Figure 9: Second Revision of the Assembly

The second revision, shown in Figure \*\*\*, had some significant changes from the initial sketch. The supports that were originally sketched had been changed two to 'A' frame supports with cuts made within the structural member to reduce the weight of the overall system. The 'A' frame support was made from three different members that would ultimately bolt together forming a solid support that would be

able to withstand the braking forces of the wheel. In addition, another change that was made from the first to second revision was the wheel being used. Although it was far simpler to buy something already made, due to the fact that a disk brake needed to be attached to the wheelbarrow wheel, a custom hub needed to be machined and the wheelbarrow wheel altered to accept the new hub attachment. Based on this new information, it was decided that a new flywheel would be machined. This way, attaching hubs to the flywheel would be easier since the design of the flywheel could allow for simple implementation. In addition, a smaller drive wheel was added to the assembly attached to a sliding linkage. Although not displayed in the solid model, a electric motor would be attached to the sliding linkage to power the drive wheel.

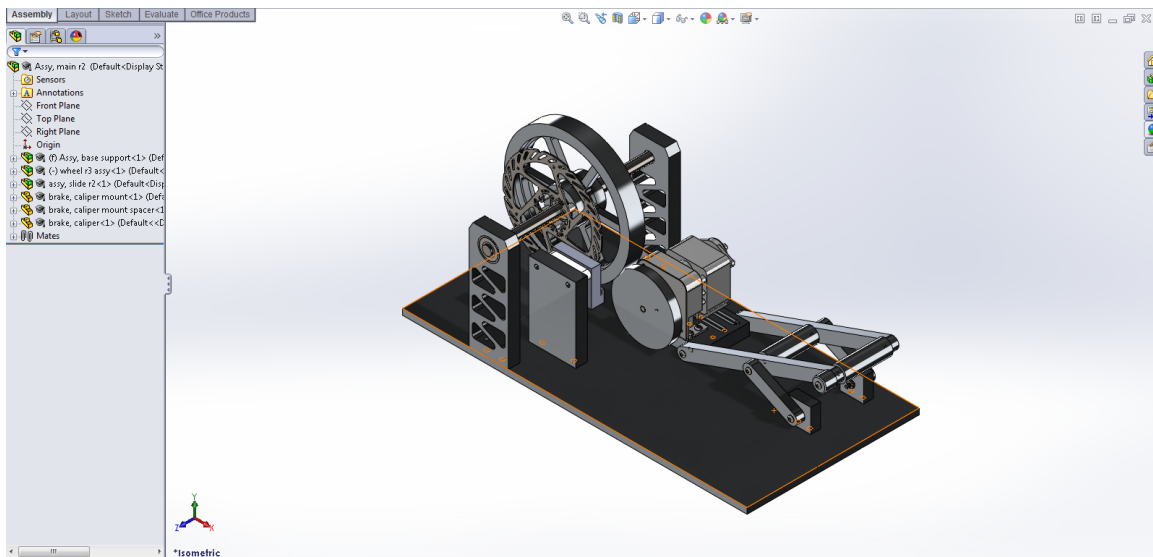


Figure 10: Third and Final Revision of the Assembly

After careful design reviews and calculations, Figure \*\*\*, was the final revision of the assembly. Two of the most distinctive changes from the second revision to the final revision are the 'A' frame supports and the mechanism that is

used to power the drive wheel. After bending and deflection simulations, the side supports of the 'A' frame design were deemed not needed. By removing the four supports, the overall weight of the assembly dropped by over 2 lbs. The main reason why the mechanism that was to power the flywheel changed so much was because of the motor that was specified. After an exhaustive search for a simple 120V AC motor, the motor shown in the final assembly was the smallest and the fastest; all other motors were either DC or required multiphase electricity. Due to its relatively large size to the assembly, it became impractical to mount the motor on the linkage assembly. Instead, a motor plate was created to hold the motor while an identical sliding linkage would move the motor to engage the flywheel. In addition, by using this method, the need to both support the motor on the linkage assembly and connect the output shaft of the motor to the drive wheel from the second revision was eliminated. Furthermore, the new linkage assembly shown in the final revision was far simpler than the second revision thus leading to a significant reduction in the possibility for the linkage assembly to both seize and fail. Finally, stress and deflection simulations showed that all other components within the assembly surpassed acceptable tolerances.

## Manufacturing

Although fixturing does not seem like a difficult idea, it can be very problematic for some parts. If there is insufficient fixturing or clamping forces when the material is being machined, the forces that are generated by the removal of material can cause the part to shift or even be thrown out of the machine. Too much fixturing or clamping forces can cause permanent deformation in thin wall parts. In some cases, custom fixturing is required to help hold the part in place. For example, all of the linkages that help push the motor were machined using a custom fixture. To machine the different linkages, the first machining operation was to drill the holes where the bolts would eventually go. Next, a sacrificial plate was also drilled in the same places so that the linkages could be placed on top and the holes would line up. This was done because it elevated the part out of the jaws of the vice so that the whole contour around the part could be cut without needing to remove the linkage from the vice jaws and flipped around. The problem with the removing the part from the vice jaws and flipping it around was that it was very hard to get the part lined up exactly so that the resulting cuts were flawless. The only part in the assembly that required two separate operations was the flywheel. Due to its shape and design, it would have been impossible to machine the part with only one operation. Fig 11 shows the first operation. The solid cylindrical stock is first pocketed to create the indented structure. Next, six symmetrical pocketing operations created the spokes of the wheel. These six pocketing operations went past the prescribed depth so that during the last operation, the part would come out as desired. The final operation first involved flipping the flywheel stock so that the

backside was being machined. Unlike the other side, this side of the flywheel only had one operation, the large pocketing operation that created the indented feature. The reason why the original six pockets were not done half way through the part was so that when the second side was being machined, the operator would not have to painstakingly align the spokes to the correct orientation.

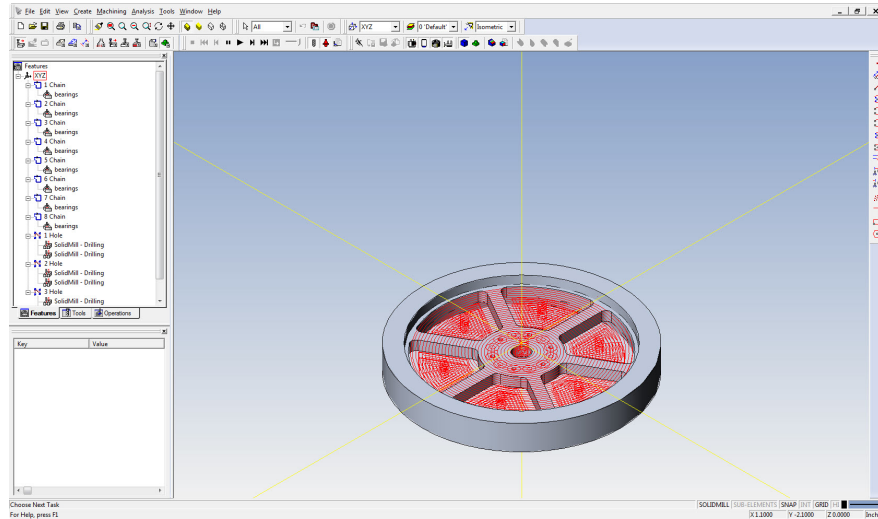
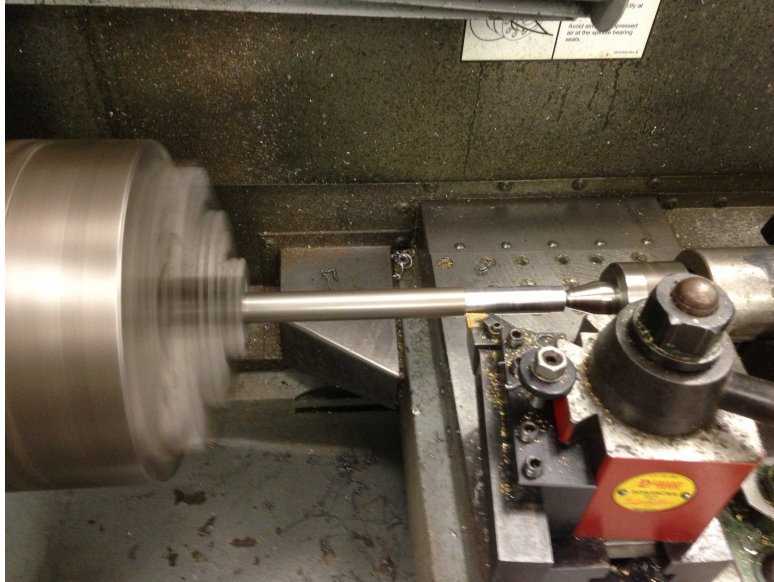


Figure 11: Flywheel Operation 1 and 2 in Esprit

Other considerations for fixturing a part are to ensure that the vibrational forces do not cause chatter in the part. Chatter is an uneven wave like pattern on the surface of a part due to vibration from the cutting end mill to the part. This problem is especially prevalent when machining long parts. When making the shaft that supported the flywheel, to help reduce the effects of chatter and deflection, a tailstock was used. A tailstock holds the work piece from the opposite side of the spindle to help reduce deflection when machining and to prevent vibrations from causing the chatter. When the part was inspected after machining, it was apparent that the shaft was not completely smooth. However, this was not due to the chatter

that was previously described; instead, this was mostly due to a chip in the cutting insert.



**Figure 12: Cutting the Shaft and Using a Tail-Stock**

When manufacturing parts for any application, tolerances are paramount, especially when parts need to fit together in an assembly. One common mistake made when designing a shaft and hole is that the designer will create the hole that the shaft is supposed to go in the exact same size as the shaft itself; this is a common error and will lead to many problems if it is not caught. When creating dimensions and tolerances for parts that need to be pressed into each other, a force fit is required. For example, the bearings into the support structure would required a force fit.

“Force fits: (FN) Force or shrink fits constitute a special type of interference fit, normally characterized by maintenance of constant bore pressures throughout the range of sizes. The interference therefore varies almost

directly with diameter, and the difference between its min and max value is small, to maintain the resulting pressures within reasonable limits.

These fits are described as follows:

FN1: Light drive fits are those requiring light assembly pressures and produce more or less permanent assemblies. They are suitable for thin sections or long fits or in cast-iron external members

FN2: Medium drive fits are suitable for ordinary steel parts, or for shrink fits on light sections. They are about the tightest fits that can be used with high-grade cast-iron external members

FN3: Heavy drive fits are suitable for heavier steel parts or for shrink fits in medium sections.

FN4 and FN5 Force fits are suitable for parts that can be highly stressed or for shrink fits where the heavy pressing forces required are impractical”<sup>1</sup>

For the bearings, a FN2 tolerance was used when determining what size to make the pocket on the support structure to ensure that the force fit would not cause the bearing to seize. In the software that was used to generate the code for the CNC machine, a 0.004” wall tolerance was used which meant that the pocket was actually enlarged by 0.004” on the diameter. The reason why the program says 0.004” and not 0.002” is because the tool that was being used to cut this pocket was measured to be another 0.002” undersized. When the part was finally machined, the bearing slid into the pocket half of the way and required a light press to seat the bearing fully into the hole.

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<sup>1</sup>Oberg, Erik. *Machinery's Handbook*. New York: Industrial, 2008. Print.



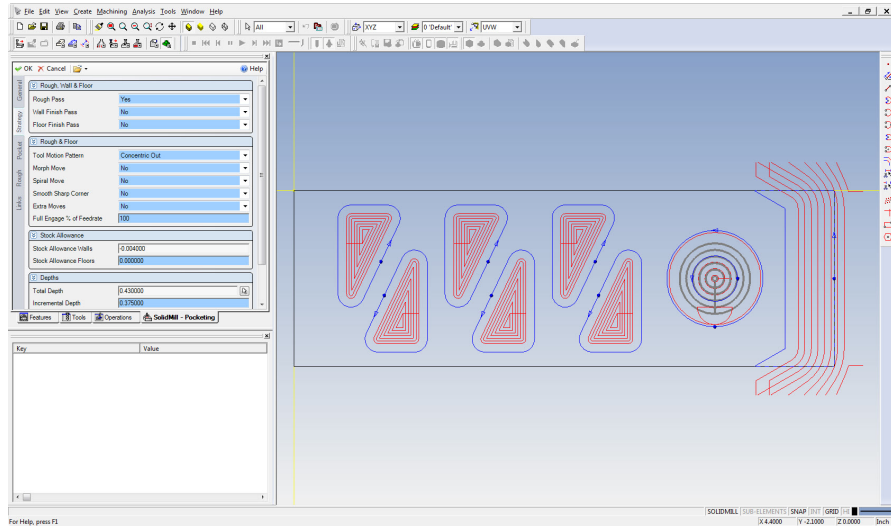


Figure 13: Support structure in Esprit

The second part on the assembly that required a specific tight tolerance was the shaft and the bearing hole. For repair and modularity reasons, the tolerance used on the shaft was a running and sliding fit.

“Running and Sliding Fits: (RC) Running and sliding fits, for which limits of clearance are given in table 8a are intended to provide a similar running performance, with suitable lubrication allowance, throughout the range of sizes. The clearance for the first two classes, used chiefly as slide fits, increase more slowly with the diameter than for the other classes, so that accurate location is maintained even at the expense of free relative motion. These fits may be described as follows:

RC1: Close sliding fits are intended for the accurate location of parts that must be assembled without perceptible play

RC2: Sliding fits are intended for accurate location, but with greater maximum clearance than class RC 1. Parts made to this fit move and turn

easily but are not intended to run freely, and in the larger sized may seize with small temperature changes.

RC3: Precision running fits are about the closest fits that can be expected to run freely and are intended for precision work at slow speeds and light journal pressures, where accurate location and minimum play are desired.

RC 4: Close running fits are intended chiefly for running fits on accurate machinery with moderate surface speeds and journal pressures, where accurate location and minimum play are desired.

RC 5 and RC 6: Medium running fits are intended for higher running speeds or heavy journal pressures, or both

RC 7: Free running fits are intended for use where accuracy is not essential, or where large temperature variations are likely to be encountered, or under both these conditions.

RC 8 and RC9: Loose running fits are intended for use where wide commercial tolerance may be necessary, together with an allowance, on the external member”<sup>2</sup>

For the shaft and bearing interaction, a RC7 fit was used. An RC 7 fit was used because in case the wheel needed to be replaced, the fit would accommodate an easy slide that would not cause the flywheel assembly to bind with the shaft. Also, great accuracy was not needed because the overall system was designed with some tolerance to ensure that the whole assembly would run smoothly.

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<sup>2</sup>Oberg, Erik. *Machinery's Handbook*. New York: Industrial, 2008. Print.

## **Conclusion and Recommendations:**

At the conclusion of this project, a physical model was manufactured and tested that demonstrated the interaction between a mechanical force that was then translated into a hydraulic force and finally converted back to a mechanical force. What first started as a project proposal and a preliminary sketch on a piece of paper led to a preliminary computer-aided-design (CAD) model. Through a set of design iterations, the CADed design was then analyzed for natural frequency, stress concentrations, minimum safety factor, deflection, and the mechanical advantage provided through the hydraulic braking system. When operating any type of rotating machinery, it is imperative that the natural frequency of the system and the frequency at the system is operating at be calculated. If the natural frequency of the system is close to the frequency that the system is operating at, the vibrations produced will eventually tear the machine apart. The natural frequency of the shaft was calculated to be 264.4 Hz while the operational frequency was at 1.667 Hz. This means that the operating speed of the system can be increased over 150 times before the system is in danger of destructive vibrations. In addition to the natural frequency, the stress concentrations of the shaft were calculated and showed that the material chosen as the shaft had a minimum safety factor of 7.48. This number far exceeds most machinery standards and thus should ensure safe operating conditions. While using the SolidWorks simulation software to calculate the stress concentrations and the safety factor, the total shaft deformation was also calculated and the resulting model showed that the maximum deflection of the shaft was 0.083mm under normal loading conditions. Finally, the mechanical advantage of the

hydraulic brake was calculated and through testing proved to be around 8.3 which is more than enough to stop the 2 lb. flywheel.

While this model proved to meet the project objectives, there are places of improvement. One future improvement would be to purchase and install a faster motor. The faster motor will allow the flywheel to rotate faster and thus the momentum of the wheel will require additional braking force. The current motor barely spins the flywheel fast enough for the internal friction of the system to be overcome. Another area for future improvement would be the optimization of the weight and size of the flywheel so that the faster motor will not interfere with the natural frequency of the system. In addition, because this model will primarily be used as a classroom demonstration model, to help students understand what is happening from the mechanical to hydraulic interaction, a part of the hydraulic brake housing could be cut away so that the internal mechanisms are exposed. Finally, sensors could be added to both show the different forces and vibrations acting on the system as well as to confirm the calculations that were initially made. These sensors would include an accelerometer attached to the shaft to detect any vibrations and movement. In addition, accelerometers could be added to the flywheel itself to see if there are any affects produced by an unbalanced wheel.

## Appendix

### Running manual

Instructions:

1. Place model on sturdy, flat surface
2. Plug in power cord into any 120V 10A circuit
3. Place one hand on linkage handle and the other on the dead man's switch
4. Press down on the handle to slide the motor plate forward
5. While applying firm pressure to the handle, ensure that the end of the motor plate engages the limit switch
6. Press and hold the dead man's switch to activate the motor (if the motor plate or dead man's switch is disengaged, the motor will stop rotating)
7. Once the flywheel is spinning, pull the brake lever to stop the rotation
8. It is also possible to spin the flywheel by hand and use the brake lever to stop the flywheel

## Bill of Materials

Description	Part Number	Vendor	Quantity
Motor	6142K73	McMaster	1
Limit Switch	7090K41	McMaster	1
Momentary Switch	6749K25	McMaster	1
Hydraulic Brake	Elixir 3	Avid	1

Mathcad File

## Weighted diameter

Weighing of each Diameter

$$d_1 := \frac{1}{11.25} = 8.889 \cdot \%$$

$$d_2 := \frac{.12}{11.25} = 1.067 \cdot \%$$

$$d_3 := \frac{10.13}{11.25} = 90.044 \cdot \%$$

$$d_1 + d_2 + d_3 = 100 \cdot \%$$

Weighed diameters

$$d_1 \cdot .75\text{in} = 0.067 \cdot \text{in}$$

$$d_2 \cdot .559\text{in} = 5.963 \times 10^{-3} \cdot \text{in}$$

$$d_3 \cdot .575\text{in} = 0.518 \cdot \text{in}$$

Weighed Total

$$d_1 \cdot .75\text{in} + d_2 \cdot .559\text{in} + d_3 \cdot .575\text{in} = 0.59 \cdot \text{in}$$

## Natural Frequencies On Center

Modulus of Elasticity

$$E_{\text{steel}} := 30457924.91 \text{psi} = 3.046 \times 10^4 \cdot \text{ksi}$$

Weighted Average Diameter

$$D_{\text{shaft}} := .590963 \text{in} = 0.591 \cdot \text{in}$$

Area Second Moment of Inertia

$$I_{\text{shaft}} := \frac{\pi \cdot D_{\text{shaft}}^4}{64} = 2.887 \times 10^{-7} \text{ft}^4$$

Mass

$$\text{Mass} := .86 \text{lb} = 0.86 \text{lb}$$

Length of Shaft

$$L_{\text{shaft}} := 11.25 \text{in} = 11.25 \cdot \text{in}$$



## Natural Frequency of Shaft

$$\omega_{\text{shaft}} := \left( \frac{2}{\pi} \right) \left( \frac{3 \cdot E_{\text{steel}} \cdot I_{\text{shaft}}}{\text{Mass} \cdot L_{\text{shaft}}^3} \right)^{\frac{1}{2}} = 264.4 \cdot \text{Hz}$$

## RPM of Motor

$$\text{RPM}_{\text{motor}} := 100 = 100$$

## Frequency of Motor

$$\omega_{\text{motor}} := \frac{\text{RPM}_{\text{motor}}}{60\text{sec}} = 1.667 \cdot \text{Hz}$$

## Frequency comparison

$$\frac{\omega_{\text{shaft}}}{\omega_{\text{motor}}} = 158.64$$

$$1 \cdot \omega_{\text{shaft}} = 158.64 \cdot \omega_{\text{motor}}$$

## Mechanical Advantage

### Torque from wrench

$$F_{\text{out}} := 30\text{ft} \cdot 1\text{b} = 30\text{ft} \cdot 1\text{b}$$

### Weight \* diameter of rotor

$$F_{\text{in}} := 13.75\text{lb} \cdot 0.262467\text{ft} = 3.609\text{ft} \cdot 1\text{b}$$

### Mechanical Advantage

$$\text{AMA} := \frac{F_{\text{out}}}{F_{\text{in}}} = 8.313$$

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